

# Quantum Gravity at the Turn of the Millennium

Gary T. Horowitz

*Physics Department*

*University of California*

*Santa Barbara, CA 93111 USA*

## **Abstract**

A very brief review is given of the current state of research in quantum gravity. Over the past fifteen years, two approaches have emerged as the most promising paths to a quantum theory of gravity: string theory and quantum geometry. I will discuss the main achievements and open problems of each of these approaches, and compare their strengths and weaknesses.

Talk presented at the *Ninth Marcel Grossmann Meeting* (MG9) Rome, July, 2000

# 1 Introduction

Two of the greatest achievements of physics in the last century were Einstein's general theory of relativity and quantum theory. Each of these theories has been extremely well tested and has been very successful. However, they are mutually incompatible. Thus our basic understanding of nature is not only incomplete – but inconsistent. One clearly needs a new theory, quantum gravity, which incorporates the principles of both of these theories and reduces to them in appropriate limits.

At first sight, the problem of constructing a quantum theory of gravity sounds easy since there are no experimental constraints! The task is simply to find any theory which unifies general relativity and quantum theory. However, on second thought, the problem sounds extremely difficult. General relativity teaches us that gravity is just a manifestation of the curvature of space and time. So quantum gravity must involve the quantization of space and time, something we have no previous experience with.

Surprisingly, even though there are no experimental constraints, this is a constraint on quantum gravity which was found in the early 1970's by studying black holes. Motivated by the close analogy between the laws of black hole mechanics and ordinary thermodynamics, Bekenstein proposed that black holes have an entropy proportional to their horizon area  $A$  [1]. Then Hawking showed that if matter is treated quantum mechanically (but gravity remains classical), black holes emit thermal radiation with a temperature  $T = \hbar\kappa/2\pi$  where  $\kappa$  is the surface gravity of the black hole [2]. This confirmed Bekenstein's ideas and fixed the coefficient:

$$S_{bh} = \frac{A}{4G\hbar} \tag{1}$$

This is an enormous entropy – much larger than the entropy in the matter that collapsed to form the black hole. In a more fundamental statistical description, the entropy should be a measure of the log of the number of accessible states. So a constraint on any candidate quantum theory of gravity is to show that the number of quantum states associated with a black hole is indeed  $e^{S_{bh}}$ . We will see that there has been considerable progress in satisfying this constraint recently.

Over the years, there has been much discussion of possible consequences of quantum gravity. Let me comment on a few of the most popular:

- *Quantum gravity will smooth out spacetime singularities*

This is false as stated. Quantum gravity cannot smooth out all singularities. Some

timelike singularities must remain, such as the one in the  $M < 0$  Schwarzschild solution. This can be seen from the following simple argument [3]. Since quantum gravity must reduce to general relativity for weak fields (and large number of quanta), there must be solutions which look like  $M < 0$  Schwarzschild at large radii, for any  $M$ . At small radii, the curvature becomes strong and the solution may be significantly modified. But if it is not singular in some sense, it would represent a state in the theory with negative total energy. Since the energy is unbounded from below, there would be no stable vacuum state.

- *Quantum gravity will allow the topology of space to change*

This is almost certainly true. There are semi-classical calculations of pair creation of magnetically charged black holes in a background magnetic field [4]. One can reliably calculate this rate for black holes with size much larger than the Planck scale. It is extremely small, but it does change the topology of space from  $R^3$  to  $R^3$  with an  $S^2 \times S^1$  wormhole attached. This is because the black holes are created with their horizons identified. It is interesting to note that if one compares the rate of black hole pair creation to the rate of magnetic monopole creation, one sees an enhancement in the black hole case of  $e^{S_{bh}}$  in line with the expectation that there are  $e^{S_{bh}}$  different species of black holes [5]. We will see further evidence for topology change shortly.

- *Black hole evaporation will violate quantum mechanics: pure states will evolve to mixed states*

Since black holes can be formed from matter in a pure state and the radiation emitted is thermal (in the semiclassical approximation) it was thought that black hole evaporation would cause pure states to evolve into mixed states. There is recent evidence (discussed below) that this is false. In a more exact treatment there are likely to be correlations between the radiation emitted at early time and later time which ensure that the evolution is unitary.

- *Space and time will not be fundamental, but derived properties.*

This is likely to be true, but the key question is, what replaces them?

Over the past fifteen years, there has been significant progress in two different approaches to quantum gravity: string theory and quantum geometry. String theory attempts to provide a unified theory of all forces and particles as well as a quantum theory of gravity. Quantum geometry attempts to quantize general relativity (by itself) in a background independent, non-perturbative way. It is well known that the usual perturbative approach to quantizing

general relativity fails since the theory is not renormalizable. But quantum general relativity may still exist nonperturbatively. Due to lack of time, I will not attempt to describe these approaches in detail. (The details keep changing anyway.) Instead, I will try to summarize the current status of each approach and some of the results that have been achieved so far.

Note on references: The list of references is necessarily incomplete and somewhat subjective. I have tried to include at least some of the key papers describing recent developments in each approach.

## 2 String theory

String theory<sup>1</sup> starts with the idea that all elementary particles are not pointlike, but excitations of a one dimensional string. If one quantizes a free relativistic string in flat spacetime one finds a infinite tower of modes of increasing mass. There is a massless spin two mode which is identified with a linearized graviton. Next, one postulates a simple splitting and joining interaction between strings and finds, remarkably, that this reproduces the perturbative expansion of general relativity. One then adds fermionic degrees of freedom to the string so the theory is supersymmetric. This makes the theory better behaved and calculations easier to control. The consistent quantization of a string turns out to impose a constraint on the spacetime dimension. In most cases, one needs ten spacetime dimensions. Contact with observations obviously requires that six of these dimensions are unobservable. The simplest possibility is the old Kaluza-Klein idea that these six dimensions are wrapped up in a small compact manifold<sup>2</sup>. The natural size of this compact manifold is the string length, a new dimensionful parameter in the theory set by the string tension. The string length,  $\ell_s$ , is related to the Planck length,  $\ell_p$ , by a power of the (dimensionless) string coupling  $g$ . In ten dimensions,  $\ell_p = g^{1/4}\ell_s$ .

Since the string represents fluctuations about the background spacetime it is propagating in, the above description is strictly perturbative. String theory has now progressed far beyond this perturbative beginning. The current status of string theory is roughly the following.

1. Classical theory is well understood. The classical equations resemble general relativity with an infinite series of correction terms involving higher powers and derivatives of the curvature. Since the correction terms become important only when the curvature is of

---

<sup>1</sup>A good general reference for string theory including many of the results discussed in this section is [6].

<sup>2</sup>Other possibilities are also being explored [7, 8].

order the string scale, any solution to general relativity with curvature smaller than  $\ell_s^{-2}$  is an approximate classical solution to string theory. To obtain exact solutions, it is useful to note that the classical field equations arise from the vanishing of the conformal anomaly in a certain two dimensional field theory. Several classes of exact solutions are known (often using special properties to ensure that the higher order correction terms vanish). One class, based on supersymmetry, includes compact Ricci flat six manifolds known as Calabi-Yau spaces which can be used to compactify spacetime down to four dimensions. Other classes include exact plane waves and group manifolds.

2. Quantum perturbation theory is well understood and well behaved. In particular, it is finite order by order in the loop expansion. Thus string theory provides a perturbatively finite quantum theory of gravity. However, the perturbation theory does not determine the quantum theory uniquely. Nonperturbative effects are important. This is evident in the fact that the loop expansion does not converge.
3. Some nonperturbative properties are known. These are mostly through clever use of supersymmetry, which guarantees that certain properties which are valid at weak coupling must continue to hold at strong coupling. In particular, extended objects (called branes) play an important role in the theory. At the perturbative level, there are five different string theories in ten dimensions which differ in the amount of supersymmetry and fundamental gauge groups they contain. There is convincing evidence that the strong coupling limit of one theory is equivalent to the weak coupling limit of another theory. These are known as duality symmetries. In fact, it is now believed that all of these theories can be obtained from a single eleven dimensional theory<sup>3</sup>.
4. There exists a complete nonperturbative formulation of the theory for certain boundary conditions. This will be discussed further below.

Up until five years ago, the status of string theory was essentially just the first two points above. Point three has an interesting consequence. Since string theory includes gravity and strong coupling implies strong gravitational fields, one might have expected the strong coupling limit of string theory to have large fluctuations of space and time, i.e., spacetime foam. But this does not seem to be the case. It appears that the strong coupling limit of

---

<sup>3</sup>This is often called M-theory, but I will continue to refer to this approach as string theory.

the theory can be described in terms of a weakly coupled theory in new variables. There is no evidence for spacetime foam.

Now I turn to discuss some results in string theory.

A) *Singularities*: The first thing to note is that the definition of a singularity is different in string theory than in general relativity, even classically. In general relativity, we usually define a singularity in terms of geodesic incompleteness which is based on the motion of test particles. In string theory, we must use test strings. So a spacetime is considered singular if test strings are not well behaved<sup>4</sup>. It turns out that some spacetimes which are singular in general relativity are completely nonsingular in string theory. A simple example is the quotient of Euclidean space by a discrete subgroup of the rotation group. The resulting space, called an orbifold, has a conical singularity at the origin. Even though this leads to geodesic incompleteness in general relativity, it is completely harmless in string theory. This is essentially because strings are extended objects.

The orbifold has a very mild singularity, but even curvature singularities can be harmless in string theory. As mentioned above, string theory has exact solutions which are the product of four dimensional Minkowski space, and a compact Calabi-Yau space. A given Calabi-Yau manifold usually admits a whole family of Ricci flat metrics. So one can construct a solution in which the four large dimensions stay approximately flat and the geometry of the Calabi-Yau manifold changes slowly from one Ricci flat metric to another. In this process the Calabi-Yau space can develop a curvature singularity. In many cases, this can be viewed as arising from a topologically nontrivial  $S^2$  or  $S^3$  being shrunk down to zero area. It has been shown that when this happens, string theory remains completely well defined. The evolution continues through the geometrical singularity to a nonsingular Calabi-Yau space on the other side [9, 10].

The reason this happens is roughly the following. There are extra degrees of freedom in the theory associated with branes wrapped around topologically nontrivial surfaces. As long as the area of the surface is nonzero, these degrees of freedom are massive, and it is consistent to ignore them. However when the surface shrinks to zero volume these degrees of freedom become massless, and one must include them in the analysis. When this is done, the theory is nonsingular.

The above singularities are all in the extra spatial dimensions. However other singu-

---

<sup>4</sup>Strictly speaking, one should also require that the other extended objects in the theory – branes – have well behaved propagation.

larities which involve time in a crucial way have also been shown to be harmless. Putting many branes on top of each other produces a gravitational field which often has a curvature singularity at the location of the brane. It has been shown that one can understand physical processes near this singularity in terms of excitations of the branes.

Despite all this progress, we still do not yet have an understanding of the most important types of singularities: those arising from gravitational collapse or cosmology.

B) *Topology change*: It has been shown unambiguously that the topology of space can change in string theory. In fact, when one evolves through a singular Calabi-Yau space as described above, the topology of the manifold changes [9, 11]. A simpler example of topology change is the following. Consider one direction in space compactified to a circle. If one identifies points under a shift  $\theta \rightarrow \theta + \pi$ , one obtains a circle of half the radius. If one identifies points under a reflection about a diameter, one obtains a line segment. It turns out that for a circle whose radius is the string scale, one can show these two  $Z_2$  actions are equivalent in string theory<sup>5</sup>. There is no way for strings to distinguish them. So one can start with one direction compactified on a large circle, slowly shrink it down to the string scale, replace it with a line segment, and then slowly expand the line segment. As far as strings are concerned, the evolution is completely nonsingular.

C) *Black hole entropy*: By far the most important result is that it has been shown that string theory can satisfy the black hole constraint mentioned earlier. For a large class of extreme and near extreme charged black holes, one can count the number of quantum string states at weak coupling with the same mass and charge as the black hole. The answer turns out to agree exactly with the prediction made by Bekenstein and Hawking [12, 13]. It is important to note that it is not just one number being reproduced. One can consider black holes with several different types of charges and angular momentum. String theory correctly reproduces the entropy as a function of all of these parameters.

The calculations are quite remarkable since one starts at weak coupling where gravitational effects are turned off and spacetime is flat. One considers configurations of branes and strings with appropriate charges and counts the number of states with a given energy. One then increases the string coupling. The gravitational field of the branes and strings becomes stronger and they eventually form a black hole. One compares the Bekenstein-Hawking entropy of the resulting black hole and finds complete agreement with the log of the number

---

<sup>5</sup>Usually, compactifying on a circle produces a  $U(1)$  gauge field. At this special radius, there is an enhanced  $SU(2)$  symmetry and these two  $Z_2$  actions are conjugate subgroups.

of states computed in flat spacetime. It is possible to do this calculation exactly only for extremal and near extremal black holes. For more general black holes, one can show that the log of the number of string states is proportional to the area [14], but the coefficient is difficult to calculate.

In some cases, one can calculate the spectrum of Hawking radiation in string theory and show that it agrees with the semiclassical calculation. This is remarkable since the spectrum is not exactly thermal, but has grey body factors arising from spacetime curvature outside the horizon. These are correctly reproduced in string theory, even though the string calculation is done in flat space. The calculations look completely different, but the results agree.

D) *Nonperturbative formulation*: By studying these black hole results more closely, people were led to a new and more complete formulation of the theory.

**AdS/CFT Conjecture** (Maldacena [15]): String theory on spacetimes which asymptotically approach the product of anti de Sitter (AdS) and a compact space, is completely described by a conformal field theory (CFT) “living on the boundary at infinity”.

In particular, string theory with  $AdS_5 \times S^5$  boundary conditions is described by a four dimensional supersymmetric  $SU(N)$  gauge theory. Since the gauge theory is defined non-perturbatively, this is a nonperturbative and (mostly) background independent formulation of string theory. A background spacetime metric only enters in the boundary conditions at infinity.

At first sight this conjecture seems unbelievable. How could an ordinary field theory describe all of string theory? I don’t have time to describe the impressive body of evidence in favor of this correspondence which has accumulated over the past few years. In the past three years, more than a thousand papers have been written on various aspects of this conjecture. A good review is [16].

This conjecture provides a “holographic” description of quantum gravity in that the fundamental degrees of freedom live on a lower dimensional space. The idea that quantum gravity might be holographic was first suggested by ’t Hooft [17] and Susskind [18] motivated by the fact that black hole entropy is proportional to its horizon area. It also confirms earlier indications that string theory has fewer fundamental degrees of freedom than it appears in perturbation theory. This conjecture provides an answer to the longstanding question raised



in the introduction: If space and time are not fundamental, what replaces them? Here the answer is that there is an auxiliary spacetime metric which is fixed by the boundary conditions at infinity. The CFT uses this metric, but the physical spacetime metric is a derived quantity. The dictionary relating spacetime concepts in the bulk and field theory concepts on the boundary is very incomplete, and still being developed.

This conjecture has an interesting consequence. Consider the formation and evaporation of a small black hole in a spacetime which is asymptotically  $AdS_5 \times S^5$ . By the AdS/CFT correspondence, this process is described by ordinary unitary evolution in the CFT. So black hole evaporation does not violate quantum mechanics. This is the basis for my earlier comment that the belief that black hole evaporation is not unitary is probably false.

### 3 Quantum Geometry

Quantum geometry<sup>6</sup> begins by rewriting (four dimensional) general relativity in first order form in terms of a tetrad and connection. But instead of the usual Lorentz connection, one uses a self dual connection. One then casts the theory into canonical form. There are the usual Hamiltonian and momentum constraints associated with diffeomorphism invariance, and a new Gauss' law constraint associated with gauge transformations. Following a standard procedure for quantizing a system with constraints, one attempts to quantize the theory by requiring that the constraint operators annihilate the physical states. The theory is analogous to an  $SU(2)$  Yang-Mills theory with an unusual Hamiltonian. However, an important difference with ordinary Yang-Mills theory is that there is no background spacetime metric. One must quantize the theory in a diffeomorphism invariant way. It turns out that there is a natural diffeomorphism invariant measure on the space of connections<sup>7</sup> which can be used to define an inner product. It is sometimes convenient to work in a loop representation in which states are functionals of loops. The relation to the connection representation is roughly  $\tilde{\psi}[\gamma] = \int DA W_\gamma[A] \psi[A]$  where  $\gamma$  is a loop and  $W_\gamma[A] = Tr P \exp \oint_\gamma A$  is the Wilson loop.

The current status of the quantum geometry approach is roughly the following:

1. There is a detailed and well defined theory of quantum geometry with a Hilbert space

---

<sup>6</sup>For a general review of this approach, see [19].

<sup>7</sup>More precisely, there is a diffeomorphism invariant measure on the space of generalized connections mod gauge transformations. A generalized connection is a map from line segments (edges) to the group.

of states, and operators describing geometrical quantities such as areas, volumes, etc. This can be viewed as the kinematical Hilbert space for the gravitational field. (The Hamiltonian constraint has not yet been imposed.) It turns out that the fundamental excitations of the geometry are one dimensional, created by gravitational analogues of Wilson loops. An orthonormal basis for this space is given by spin networks, which are, roughly, graphs whose edges are labeled with representations of  $SU(2)$ .

2. Progress has been made in understanding the dynamics, i.e., the Hamiltonian constraint. (This is much more difficult than the other constraints since it is quadratic in momenta and needs to be regulated.) For example, a rigorously defined and finite Hamiltonian operator has been constructed and states have been found which are annihilated by this operator [20]. This defines a consistent generally covariant four dimensional quantum field theory, but it may not reduce to general relativity in the classical limit [21]. Also there has been progress in a path integral approach, in which the spin networks are generalized to “spin foams” which are two dimensional surfaces glued together along edges [22].
3. There has been recent progress in constructing kinematical semi-classical states [23]. These are quantum states which approximate classical spacetimes with minimum uncertainty. They are important for gaining a physical interpretation of quantum states, and are needed as a first step to studying scattering in this approach.
4. Although supersymmetry does not seem to be necessary in this approach, there has been progress in extending this approach to supergravity [24].

Let me now describe some of the results in the quantum geometry approach.

A) *Discreteness*: The geometric operators describing areas and volumes have a purely discrete spectrum. For example, the area of a small surface crossing an edge of a spin network is directly related to the  $SU(2)$  representation on the edge. The area of a large surface is just the sum of the contributions from each edge of the spin network it crosses. Since all geometric operators have a discrete spectrum, geometry is really quantized. The usual continuum picture is only a coarse grained approximation. This is another answer to the question of what replaces our usual notions of space and time, if they are not fundamental.

B) *Black hole entropy*: One can reproduce the entropy of all nonrotating black holes (including Schwarzschild) by counting quantum states [25]. The answer agrees with the

Bekenstein-Hawking prediction up to a single undetermined dimensionless constant, called the Immirzi parameter. This parameter arises since there are inequivalent ways of quantizing the classical phase space [26]. If you fix this parameter to give the right answer for Schwarzschild, one automatically gets the right answer for all charged black holes. This is different from the situation in string theory, in which current calculations only gives the entropy of a general black hole up to an undetermined factor of order one which is not simply related to a parameter in the theory.

One might wonder how one can count physical states of the black hole if one does not yet have complete control over the Hamiltonian constraint. The answer is the following. If one starts with the Einstein action in appropriate variables defined outside a stationary black hole, one can show that one needs to add a surface term at the horizon which is essentially a  $U(1)$  Chern-Simons action (recall that we are in  $3 + 1$  dimensions). Physically, this Chern-Simons action describes fluctuations of the horizon geometry. The Hilbert space consists of states in the bulk and states on the horizon, coupled in a well defined way. What one actually counts are states of the boundary Chern-Simons theory, on a sphere with certain punctures. The horizon area is kept fixed and corresponds to a sum of contributions associated with each puncture. One then assumes that each of these boundary states can be connected to a bulk state satisfying the Hamiltonian constraint.

C) *Area eigenvalues are consistent with Hawking radiation:* In this framework, Hawking radiation can be thought of as analogous to atomic transitions: the area drops to a lower eigenstate, and one emits a quanta of energy. But to reproduce an approximately thermal spectrum at low frequency, it is important that the area eigenvalues are not evenly spaced [27]. This can easily be seen as follows. Suppose the area eigenvalues were  $A_n \sim n$  in Planck units. Even though the Planck length is so small, this would lead to observable effects. Since  $A \sim M^2$ ,  $\delta A \sim M \delta M$ . So  $\delta M \sim 1/M = \omega_0$ . Thus black hole radiation could only consist of particles with energy  $\omega_0$ ,  $2\omega_0$ ,  $3\omega_0$ , etc. But the Hawking temperature of the black hole is of order  $\omega_0$ , so this should be the peak of the thermal spectrum. Fortunately, one finds that the level spacing between area eigenvalues goes to zero very rapidly,  $A_n - A_{n-1} \sim e^{-\sqrt{A_n}}$ , which is consistent with a thermal spectrum even at low frequency.

The quantum geometry approach has recently made contact with another approach to quantizing general relativity, based on the close analogy with topological field theory. The basic idea is the following. Consider a theory in  $D$  dimensions of a gauge field  $A_\mu^a$  with

gauge group  $G$  and  $D - 2$  form taking values in the (dual of the) adjoint representation of  $G$ ,  $B_{\mu\cdots\nu a}$ . The action is simply  $S_{BF} = \int F \wedge B$  where  $F$  is the usual field strength of  $A$ . This is a topological field theory, which is independent of a spacetime metric. It is known in the literature as BF theory [28]. This theory can be quantized by path integral or canonical methods. For  $D = 3$  and  $G = SO(2, 1)$ , this action is precisely 2 + 1 general relativity, where  $B$  is interpreted as a triad of orthonormal vectors. Even in higher dimensions, general relativity is equivalent to  $S_{BF}$  plus a constraint. This can be seen by writing the Einstein action in first order form

$$S = \int R^{ab} \wedge e^c \cdots \wedge e^d \epsilon_{abc\cdots d} \quad (2)$$

where  $R^{ab}$  is the curvature two form of an  $SO(D - 1, 1)$  connection and  $e^a$  is a set of  $D$  orthonormal vectors. This is clearly equivalent to  $S_{BF}$  with gauge group  $G = SO(D - 1, 1)$  and  $B$  constrained to take the form of the wedge product of vectors. In any dimension, this constraint can be written as a quadratic condition on  $B$  [29]. So general relativity is equivalent to a topological field with a simple quadratic constraint!

Returning to four dimensions, a functional integral approach to quantization has been developed starting with a simplicial decomposition of the four manifold  $M$ . The two form  $B$  is defined on the faces, and the connection is defined on the edges of a dual triangulation, in which each  $i$ -simplex is replaced by a  $(4-i)$ -simplex. Evaluating the functional integral involves summing over group representations, and the constraint on  $B$  can be simply implemented by restricting the group representations one must sum over [30]. The dual triangulation involves two dimensional faces joined at edges, and one can show that this leads to the same description as the “spin foam” model mentioned earlier [31]. The spin foam model was originally obtained in a completely different way, by extending the spin networks of the canonical theory to a four dimensional framework. A topological theory can be completely described by a single triangulation, but general relativity requires a sum over triangulations. A field theory formulation has been found in which the Feynman diagrams are in one-to-one correspondence with the spin foams [32]. So summing over Feynman diagrams naturally sums over spin foams, which is like summing over four geometries. This is analogous to the matrix theory description of two dimensional gravity. Although much of the work in this direction has been restricted to Euclidean general relativity (where the group is compact), there has been recent work in extending this to Lorentzian general relativity.

## 4 Comparisons

From the mid 1980's to the mid 1990's, a key difference between string theory and quantum geometry was that string theory was inherently perturbative and quantum geometry was not. (Indeed, the latter was often called “nonperturbative quantum gravity” to distinguish it from string theory.) As I have tried to emphasize, this is no longer the case. There is now a complete nonperturbative formulation of string theory (at least for certain boundary conditions).

Both string theory and quantum geometry have given strong evidence that they satisfy the black hole constraint: They can reproduce the entropy of black holes by counting quantum states. But they do so in very different ways. Quantum geometry is directly counting fluctuations of the event horizon, while string theory extrapolates the black hole to weak coupling and counts states of strings (and branes) in flat spacetime. At the moment, the string calculations give exact results (including the factor of  $1/4$ ) only for extreme and near extreme charged black holes. Rotation can be included. In contrast, the quantum geometry calculations apply to all nonrotating black holes, even those which are far from extremality. They do not recover the factor  $1/4$  uniquely, but it can be reproduced by fixing the one free parameter in the theory. In this approach, one counts horizon states and assumes that they can be extended to states satisfying the Hamiltonian constraint.

The main advantage of the quantum geometry approach is that it is directly facing the challenge of quantizing space and time. String theory has not yet done this. Initially it avoided the issue by focusing on perturbation theory. More recently, it has side-stepped the issue by using duality to relate strong coupling to weak coupling, or using holography to remove spacetime altogether. We do not yet have a good understanding of how to recover spacetime from its holographic description.

The main disadvantage of quantum geometry is that it is still trying to deal with the Hamiltonian constraint. This has always been the main difficulty of canonical quantization of general relativity, and despite enormous progress, it has not yet been resolved.

The main advantage of string theory is that it is much more ambitious. It attempts to provide not just a quantum theory of gravity but also a unified theory of all particles and forces. String theory has also achieved more, including the results on singularities and topology change mentioned earlier, and more detailed calculations of Hawking radiation.

The main disadvantage of string theory is that one has to accept a lot of extra structure:

extra dimensions, supersymmetry, extra particles. At first sight it does not seem very economical. But nature may in fact include all of these features, and it is only experimental limitations which have kept them hidden. Furthermore, we have seen evidence that string theory has fewer degrees of freedom than it appears, so it may be more economical than it looks.

## 5 Looking ahead

In the next few years, I expect to see progress in both approaches. But in the long run, things depend on the following key question:

*Does quantum general relativity exist as a consistent theory?*

If so, the quantum geometry approach will probably succeed and construct it. If not, it will fail. But even in this case, ideas from quantum geometry are likely to be useful in string theory e.g. to recover spacetime from its holographic description. If quantum general relativity exists and is (in a suitable sense) unique, then quantum geometry must be included in string theory since string theory includes general relativity.

How could quantum geometry be combined with string theory? There are already several similarities between the two approaches. For example, one dimensional objects play a key role in both. In string theory, this is the starting point: all elementary particles are excitations of a one dimensional string. In quantum geometry, one finds (at the end of a lengthy analysis) that fundamental excitations of the geometry are one dimensional. Given the AdS/CFT conjecture, there are further similarities arising from the fact that gauge theories play an important role in both approaches. String theory with  $AdS_5 \times S^5$  boundary conditions is completely described by a four dimensional  $SU(N)$  gauge theory. In the quantum geometry approach, four dimensional general relativity is described by something resembling an  $SU(2)$  gauge theory. Could the fact that  $SU(2)$  is contained in  $SU(N)$  be related to the fact that string theory describes many more fields than just four dimensional general relativity? Furthermore, in the quantum geometry approach, Wilson loops are the basic operators creating fundamental one dimensional excitations of the geometry. In string theory, Wilson loops in the boundary gauge theory describe strings in the bulk spacetime.

Of course, at the moment there are also crucial differences between these approaches. The

gauge theory in string theory is a standard Yang-Mills theory with its usual Hamiltonian and a fixed spacetime metric. In quantum geometry, there is a Hamiltonian constraint, spatial diffeomorphism constraints and no background metric. Another key difference is that one of the main predictions of the quantum geometry approach is that all areas are discrete. But in string theory we have seen that one can wrap extended objects around compact surfaces in the extra dimensions to produce new states. The mass of these states is directly proportional to the area of the surface. Supersymmetry requires that one can change this area continuously. So it appears that area is not quantized in this case. The future will tell whether, at a deeper level, these differences are superficial or fundamental.

Of course the goal of both approaches is to answer basic questions such as what was physics like at the big bang. Given the recent progress, one may be hopeful that answers will be available soon into the next millennium.

### **Acknowledgements**

It is a pleasure to thank A. Ashtekar, T. Banks, K. Krasnov, D. Marolf, J. Maldacena, J. Polchinski, C. Rovelli, and A. Strominger for improving my understanding of the topics discussed here. This work was supported in part by NSF Grant PHY-0070895.

# References

- [1] J. Bekenstein, “Black Holes and Entropy”, *Phys. Rev. D* 7 (1973) 2333.
- [2] S. Hawking, “Particle Creation by Black Holes”, *Commun. Math. Phys.* 43 (1975) 199.
- [3] G. Horowitz and R. Myers, “The Value of Singularities”, *Gen. Rel. Grav.* 27 (1995) 915, gr-qc/9503062.
- [4] D. Garfinkle and A. Strominger, “Semiclassical Wheeler Wormhole Production”, *Phys. Lett. B* 256 (1991) 146.
- [5] D. Garfinkle, S. Giddings, and A. Strominger, “Entropy in Black Hole Pair Production”, *Phys. Rev. D* 49 (1994) 958, gr-qc/9306023.
- [6] J. Polchinski, *String Theory*, in 2 vols., Cambridge Univ. Press, 1998.
- [7] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, “The Hierarchy Problem and New Dimensions at a Millimeter”, *Phys. Lett. B* 429 (1998) 263, hep-ph/9803315.
- [8] L. Randall and R. Sundrum, “An Alternative to Compactification”, *Phys. Rev. Lett.* 83 (1999) 4690, hep-th/9906064.
- [9] P. Aspinwall, B. Greene, and D. Morrison, “Calabi-Yau Moduli Space, Mirror Manifolds and Spacetime Topology Change in String Theory”, *Nucl. Phys. B* 416 (1994) 414, hep-th/9309097.
- [10] A. Strominger, “Massless Black Holes and Conifolds in String Theory”, *Nucl. Phys. B* 451 (1995) 96, hep-th/9504090.
- [11] B. Greene, D. Morrison, and A. Strominger, “Black Hole Condensation and the Unification of String Vacua”, *Nucl. Phys. B* 451 (1995) 109.
- [12] A. Strominger and C. Vafa, “Microscopic Origin of the Bekenstein-Hawking Entropy”, *Phys. Lett. B* 379 (1996) 99, hep-th/9601029.
- [13] A. Peet, “TASI Lectures on Black Holes in String Theory”, hep-th/0008241.



- [14] L. Susskind, “Some Speculations about Black Hole Entropy in String Theory”, hep-th/9309145; G. Horowitz and J. Polchinski, “A Correspondence Principle for Black Holes and Strings”, Phys. Rev. D55 (1997) 6189, hep-th/9612146; T. Damour and G. Veneziano, “Self-gravitating Fundamental Strings and Black Holes”, Nucl. Phys. B568 (2000) 93, hep-th/9907030.
- [15] J. Maldacena, “The Large N Limit of Superconformal Field Theories and Supergravity”, Adv. Theor. Phys. 2 (1998) 231, hep-th/9711200.
- [16] O. Aharony, S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, “Large N Field Theories, String Theory and Gravity”, Phys. Rept. 323 (2000) 183, hep-th/9905111.
- [17] G. 't Hooft, “Dimensional Reduction in Quantum Gravity”, gr-qc/9310026.
- [18] L. Susskind, “The World as a Hologram”, J. Math. Phys. 36 (1995) 6377, hep-th/9409089.
- [19] C. Rovelli, “Loop Quantum Gravity”, Living Reviews 1 (1998), gr-qc/9710008.
- [20] T. Thiemann, “Anomaly-free Formulation of Non-perturbative, Four-dimensional Lorentzian Quantum Gravity”, Phys. Lett. B380 (1996) 257, gr-qc/9606088.
- [21] J. Lewandowski and D. Marolf, “Loop Constraints: A Habitat and their Algebra”, Int. J. Mod. Phys. D7 (1998) 299, gr-qc/9710016.
- [22] M. Reisenberger and C. Rovelli, “Sum over Surfaces form of Loop Quantum Gravity”, Phys. Rev. D56 (1997) 3490, gr-qc/9612035.
- [23] T. Thiemann, “Gauge Field Theory Coherent States”, hep-th/0005233.
- [24] Y. Ling and L. Smolin, “Supersymmetric Spin Networks and Quantum Supergravity”, Phys. Rev. D61 (2000) 044008, hep-th/9904016; “Holographic Formulation of Quantum Supergravity”, hep-th/0009018.
- [25] A. Ashtekar, J. Baez, A. Corichi and K. Krasnov, “Quantum Geometry and Black Hole Entropy”, Phys. Rev. Lett. 80 (1998) 904, gr-qc/9710007; A. Ashtekar, J. Baez, and K. Krasnov, “Quantum Geometry of Isolated Horizons and Black Hole Entropy”, gr-qc/0005126.

- [26] C. Rovelli and T. Thiemann, “The Immirzi Parameter in Quantum General Relativity”, Phys. Rev. D57 (1998) 1009.
- [27] J. Bekenstein and V. Mukhanov, “Spectroscopy of the Quantum Black Hole”, Phys. Lett. B360 (1995) 7, gr-qc/9505012.
- [28] G. Horowitz, “Exactly Soluble Diffeomorphism Invariant Theories”, Commun. Math. Phys. 125 (1989) 417; M. Blau and G. Thompson, “Topological Gauge Theories of Antisymmetric Tensor Fields”, Ann. Phys. 205 (1991) 130.
- [29] L. Freidel, K. Krasnov, R. Puzio, “BF Description of Higher-Dimensional Gravity Theories”, hep-th/9901069.
- [30] J. Barrett and L. Crane, “Relativistic Spin Networks and Quantum Gravity”, J. Math. Phys. 39 (1998) 3296, gr-qc/9709028.
- [31] J. Baez, “An Introduction to Spin Foam Models of Quantum Gravity and BF Theory”, gr-qc/9905087; J. Barrett, “State Sum Models and Quantum Gravity”, gr-qc/0010050.
- [32] R. De Pietri, L. Freidel, K. Krasnov, and C. Rovelli, “Barrett-Crane Model from a Boulatov-Ooguri Field Theory over a Homogeneous Space”, Nucl. Phys. B574 (2000) 785, hep-th/9907154; A. Perez, “Finiteness of a Spinfoam Model for Euclidean Quantum General Relativity”, gr-qc/0011058.